Math 20550 - Summer 2016
Stokes' and Divergence Theorems Worksheet
July 18, 2016

Question 1. *Compute the line integral* $\int_C \mathbf{F}\cdot d\mathbf{r}$ *where* $\mathbf{F} = \langle y^2, -z^2, x\rangle$ *and* $\mathbf{r}(t) = \langle 3\cos t, 4\cos t, 5\sin t\rangle$, $0 \le t \le 2\pi$.

Question 2. *Compute the flux of* $\mathbf{F} = \langle x, -2y, 3z\rangle$ *where* S *is the sphere* $x^2+y^2+z^2 = 6$ *with outward orientation.*

Question 3. *Compute* $\iint_S (\nabla\times\mathbf{F})\cdot d\mathbf{S}$ *where* $\mathbf{F} = \langle 2y, -z, x-y-z\rangle$ *and* S *is the piece of the sphere* $x^2+y^2+z^2 = 25$, $3 \le x \le 5$, *which has orientation pointing in the positive* x*-direction.*

Question 4. *Compute* $\iint_S \mathbf{F}\cdot d\mathbf{S}$ *where* $\mathbf{F} = \langle z-x, x-y, 2y-z\rangle$ *and* S *is the boundary of the region between the spheres* $x^2+y^2+z^2 = 4$ *and* $x^2+y^2+z^2 = 16$.

Question 5. *Consider the argument:*
The Divergence Theorem says,

$$\iint_{\partial K} \mathbf{F}\cdot d\mathbf{S} = \iiint_K (\nabla\cdot\mathbf{F})\,dV,$$

and Stokes' theorem says,

$$\oint_{\partial \mathbf{S}} \mathbf{F}\cdot d\mathbf{r} = \iint_S (\nabla\times\mathbf{F})\cdot d\mathbf{S},$$

so I can combine the two statements to get

$$\oint_{\partial \mathbf{S}} \mathbf{F}\cdot d\mathbf{r} = \iint_S (\nabla\times\mathbf{F})\cdot d\mathbf{S} = \iiint_K (\nabla\cdot\mathbf{F})\,dV.$$

Since $\nabla\cdot(\nabla\times\mathbf{F}) = \mathrm{div}(\mathrm{curl}\ \mathbf{F}) = 0$, *this then means that the line integral of any vector field along the boundary of a surface is always* 0.

(a) *Critique the above argument.*

(b) *Assume that* $\mathbf{F}$ *is a* C^1 *vector field and compute the flux integral* $\iint_S (\nabla\times\mathbf{F})\cdot d\mathbf{S}$ *where* S *is the boundary of the cube* $[-1,1]\times[-1,1]\times[-1,1]$.

Question 6. *Compute the surface integral* $\iint_S (\nabla\times\mathbf{F})\cdot d\mathbf{S}$ *where* $\mathbf{F} = \langle yz, -xz, xy\rangle$ *and* S *is the piece of the ellipsoid* $x^2+y^2+8z^2 = 1$, $z \ge 0$ *and upward orientation. (Hint: consider an easier surface.)*